# 2017 Canadian Computing Olympiad
# Day 1, Problem 1
# **Vera and Trail Building**

**Time Limit: 1 second**

**Problem Description**

Vera loves hiking and is going to build her own trail network. It will consist of $V$ places numbered from 1 to $V$, and $E$ bidirectional trails where the $i$-th trail directly joins two distinct places $a_i$ and $b_i$. Vera would like her network to be connected so it should be possible to hike between any two places using the trails. It is possible that there could be more than one trail directly joining the same pair of places.

Vera considers two places $a, b$ with $a < b$ to form a *beautifully connected pair* if it is possible to hike using the trails from $a$ to $b$ then back to $a$ without hiking on the same trail more than once. She thinks her trail network would be beautiful if it had exactly $K$ beautifully connected pairs.

Vera does not want her network to be too large, so the network should satisfy $1 \leq V, E \leq 5000$.

Given $K$, help Vera find any beautiful trail network.

**Input Specification**

There is one line of input, which contains the integer $K$ ($0 < K \leq 10^7$).

For 3 of the 25 available marks, $K \leq 1000$.

For an additional 6 of the 25 available marks, $K \leq 10^5$.

**Output Specification**

Print a beautiful network in the following format:

- the first line should contain the number of vertices, $V$, followed by one space, followed by the number of edges, $E$;
- each of the next $E$ lines should contain two integers, $a_i$ and $b_i$, separated by one space, indicating a trail between places $a_i$ and $b_i$ ($1 \leq i \leq E$).

The trails can be printed in any order. The two places of any trail can be printed in any order. If there are multiple beautiful trail networks, print any of them. It is guaranteed that a solution always exists.

**Sample Input 1**

```
2
```

**Output for Sample Input 1**

```
4 5
1 2
2 1
3 4
4 3
1 4
```

**Explanation for Output for Sample Input 1**

The two beautifully connected pairs are $1, 2$ and $3, 4$.

**Sample Input 2**

```
6
```

**Output for Sample Input 2**

```
4 4
1 2
2 3
3 4
4 1
```

**Explanation for Output for Sample Input 1**

All pairs of places form a beautifully connected pair.

# 2017 Canadian Computing Olympiad
# Day 1, Problem 2
# **Cartesian Conquest**

**Time Limit: 2 seconds**

**Problem Description**

Long ago, in the land of Cartesia, there ruled the Rectangle Empire. The Empire was large and prosperous, and it had great success with expanding its territory through frequent conquests. The citizens of this ancient civilization followed many curious customs. Unfortunately, the significance of these are now shrouded in mystery.

The Rectangle Empire operated under a system of rectangular districts. These districts were carefully managed to meet three special criteria.

1. The Empire's territory is divided into districts such that each piece of land controlled by the Empire belongs to exactly one district.
2. The boundaries of the districts, when viewed on a map, must be rectangles such that the length of the longer side of the rectangle is twice the length of the shorter side.
3. The side lengths of the districts must be integers, when measured in $\Xi$ (note that $\Xi$ was the primary unit of length in the Rectangle Empire).

When the empire was first established, it consisted of a single district. Since then, the empire has gained additional districts through conquest of neighbouring regions. Whenever the empire gained control over a new region of land, they always established a single new district using that exact land. This means that the empire was always mindful about the geometric properties of the land they were hoping to conquer. You can assume that no two of these conquests occurred at the same time.

The addition of new districts was the only way that the boundaries of the empire ever changed. Furthermore, each district, once added, was never modified or merged with another.

The final, most important tradition of the Rectangle Empire was to make sure that the overall territory of the empire was always a rectangle, though it did not necessarily need to satisfy the 2:1 ratio for the side lengths that individual districts satisfy.

Recently, archeologists have discovered that at one point in time, the empire had dimensions $N$ by $M$ (measured in $\Xi$). You need not be alarmed if these numbers are very large; after all, Cartesia is an infinite plane. Your task is to estimate the number of districts in the empire when it was at this size. Over all possible ways that the empire was founded and expanded, what is the minimum and maximum number of districts?

## Input Specification

The input will be a single line, containing two integers $N$ and $M$ ($1 \leq N, M \leq 10^8$).

For 5 of the 25 marks available, $N, M \leq 1000$.

For an additional 8 of the 25 marks available, $N, M \leq 10^6$.

## Output Specification

Output a single line, containing the minimum number of districts, followed by a space, followed by the maximum number of districts.

## Sample Input

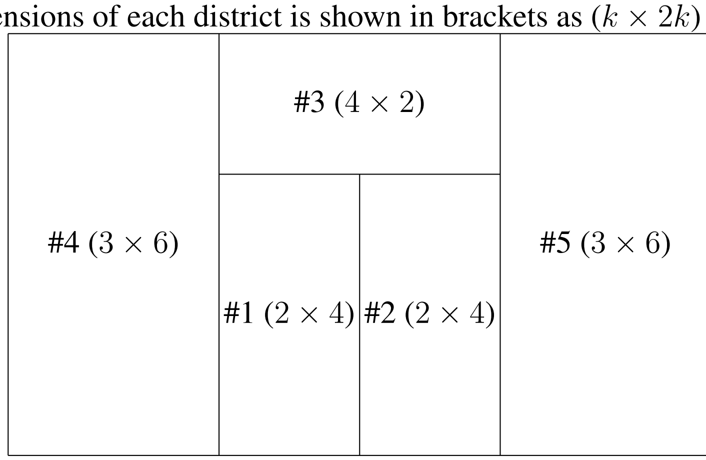
```
10 6
```

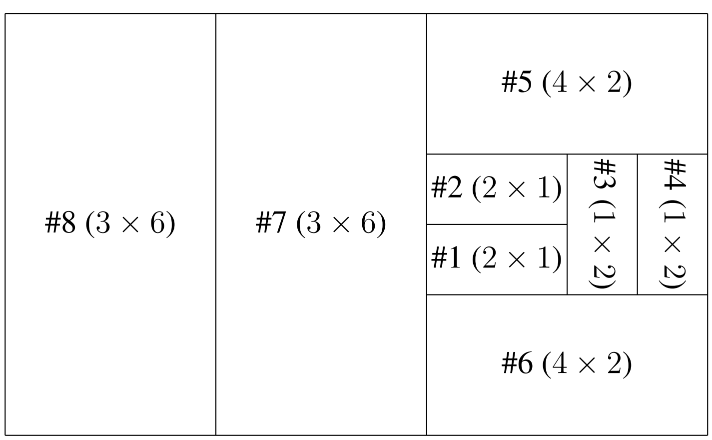
## Output for Sample Input

```
5 8
```

## Explanation of Output for Sample Input

The illustrations below show how this minimum and maximum number of districts could have been achieved. Districts are labelled #1, #2, #3, ... giving the order in which they were added to the region. The dimensions of each district is shown in brackets as $(k \times 2k)$ or $(2k \times k)$:



# 2017 Canadian Computing Olympiad
# Day 1, Problem 3
# Vera and Modern Art

**Time Limit: 4 seconds**

**Problem Description**

After being inspired by the great painter Picowso, Vera decided to make her own masterpiece. She has an empty painting surface which can be modeled as an infinite 2D coordinate plane. Vera likes powers of two $(1, 2, 4, 8, 16, \ldots)$ and will paint some some points in a repeated manner using step sizes which are a power of two.

Vera will paint $N$ times. The $i$-th time can be described by three integers $x_i, y_i, v_i$. Let $a_i$ be the largest power of two not greater than $x_i$ and let $b_i$ be the largest power of two not greater than $y_i$. Vera will add a paint drop with colour $v_i$ to all points that are of the form $(x_i + a_i p, y_i + b_i q)$, where $p, q$ are non-negative integers. A point may have multiple paint drops on it or have multiple drops of the same colour.

Then Vera will ask $Q$ questions. For the $j$-th question she wants to know the colour at the point $(r_j, c_j)$. The colour at a point is equal to the sum of the colours of all paint drops at that point. If there are no paint drops at a point, the colour of that point is $0$.

Since you are forced to be her art assistant, you will have to answer Vera's questions.

**Input Specification**

The first line contains two integers $N$, $Q$, separated by one space ($1 \leq N, Q \leq 2 \cdot 10^5$).

The next $N$ lines each contain three space-separated integers, $x_i$, $y_i$, $v_i$ representing the paint drops of colour $v_i$ ($1 \leq i \leq N$; $1 \leq v_i \leq 10\,000$; $1 \leq x_i, y_i \leq 10^{18}$).

The next $Q$ lines each contain two space-separated integers $r_j$, $c_j$, representing the $Q$ questions about the point $(r_j, c_j)$ ($1 \leq j \leq Q$; $1 \leq r_j \leq 10^{18}$; $1 \leq c_j \leq 10^{18}$).

For 5 of the 25 available marks, $N, Q \leq 2000$.

For an additional 5 of the 25 available marks, $y_i = 1$ ($1 \leq i \leq N$).

For an additional 5 of the 25 available marks, $N, Q \leq 10^5$ and $1 \leq r_j, c_j \leq 10^9$ ($1 \leq j \leq Q$).

**Output Specification**

The output will be $Q$ lines. The $j$-th line ($1 \leq j \leq Q$) should have one integer, which is the colour of point $(r_j, c_j)$.

### Sample Input

```
5 6
1 2 1
3 4 2
4 5 3
6 3 4
7 1 5
2 6
7 8
5 9
11 2
10 7
4 5
```

### Output for Sample Input

```
1
8
0
6
4
3
```

### Explanation of Output for Sample Input

Let colour $1$, $2$, $3$, $4$, $5$ be red, blue, green, orange, purple respectively.

Let $p, q$ be non-negative integers, then:

- Points $(1 + p, 2 + 2q)$ have a red paint drop.
- Points $(3 + 2p, 4 + 4q)$ have a blue paint drop.
- Points $(4 + 4p, 5 + 4q)$ have a green paint drop.
- Points $(6 + 4p, 3 + 2q)$ have a orange paint drop.
- Points $(7 + 4p, 1 + q)$ have a purple paint drop.

The painting from $(0, 0)$ to $(11, 11)$ is shown on the next page:

We can see that:

- $(2, 6)$ has a red paint drop, so it has colour $1$.
- $(7, 8)$ has a red, blue and purple paint drop, so it has colour $1 + 2 + 5 = 8$.

- $(5, 9)$ has no paint drops, so it has colour $0$.
- $(11, 2)$ has a red and purple paint drop, so it has colour $1 + 5 = 6$.
- $(10, 7)$ has a orange paint drop, so it has colour $4$.
- $(4, 5)$ has a green paint drop, so it has colour $3$.